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# Coevolutionary dynamics of opinions and networks: From diversity to uniformity

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We investigate the coevolutionary dynamics of opinions and networks based upon majority-preference (MP) and minority-avoidance (MA) rules. Under MP, individuals adopt the majority opinion among their neighbors; while in MA individuals can break the link to one holding a minority and different opinion, and rewire either to neighbors of their neighbors with the same opinion or to a random one from the whole population except their nearest neighbors. We study opinion formation as a result of combination of these two competing rules, with a parameter tuning the balance between them. We find that the underlying network can be self-organized into connected communities with like-minded individuals belonging to the same group; thus a broad variety of opinions coexist. Diverse opinions disappear in a population in which all individuals share a uniform opinion, when the model parameter exceeds a critical value. Furthermore, we show that an increasing tendency to redirect to neighbors of neighbors is more likely to result in a consensus of opinion.

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## I. INTRODUCTION

Opinion dynamics or opinion formation is a traditional research area in social science and social psychology. Recently, in the physics community, it has attracted much attention for study of collective behaviors in social and economic systems. This results in a significant overlap between social science and statistical physics, i.e., sociophysics [1]. In analogy with the Ising model [2], some models were developed to address this issue [3–8]: the voter model [3], the majority-rule model [4], and the bounded-confidence model [8], to name a few. These models assume opinions as discrete or continuous variables. Much previous work studied the opinion formation process on fixed networks and focused on how network topology affects the dynamics. More recently, instead of considering opinion dynamics on static networks, some researchers took into account the dynamics on continuously evolving networks [9–15], namely, network change in response to opinion and opinion change in response to the network. Indeed, many networks have an adaptive dynamical nature: their evolution occurs on a fast or slow time scale compared with the dynamics taking place on them. Such considerations of the entangled dynamics of opinions and networks are thus plausible and relevant. Motivated by these existing works, in this paper, we consider a simple coevolution model that combines opinion dynamics and a network rewiring process. We are interested in understanding the conditions in which the opinion evolves from diverse to uniform.

Our study is based on the majority-rule model, which assumes that individuals preferentially follow the crowd in their opinion update (e.g., herding behavior in public choices). In addition, each individual is assumed to show minority-avoidance behavior, that is, the focal individual tends to remove the link to one holding a different and minority opinion. In order to keep the total number of edges constant, a new connection is formed between the focal individual and another one chosen from the network. Here, we view the creation of new links as a balance between two opposing forces: order and randomness. Order indicates the tendency of an individual to connect to neighbors of neighbors; randomness means that a new link is formed between the focal individual and one randomly chosen from the rest of the population. We take into account this trade-off in the network rewiring process and reveal that an increasing tendency of rewiring to neighbors of neighbors is more likely to reduce the number of surviving opinions. Furthermore, we focus on the final number of existing opinions in the network as a result of the regime where opinion update or network rewiring dominates the dynamics. Interestingly, our investigation will show that the coevolving network exhibits characteristics ranging from interconnected communities (where like-minded individuals belong to the same group) to a single community (where everyone holds the same opinion).

## II. THE MODEL

Let us briefly introduce the model used in the present paper. Initially, $N$ individuals are embedded in a random regular graph in which the links denote the acquaintances between them. The total number of edges $M$ is fixed during the coevolution of opinions and networks. This constraint implies that the evolution is conservative, in the sense that individuals can maintain only a limited number of connections and the possibilities of resulting network configurations are limited. Each individual holds one of $G$ possible opinions on some topic of interest, denoted by $g_i$. At each time step, an individual is chosen at random to update either its opinion or neighborhood according to the following two rules.

(1) *Majority preference* (MP): with probability $p$, the focal individual accepts the specific opinion held by a majority

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of its neighbors (i.e., the opinion the largest number of individuals among its neighbors hold).

(2) *Minority avoidance* (MA): otherwise, with likelihood $1-p$, the focal individual removes the link to one of its neighbors who holds a minority and different opinion (i.e., the different opinion held by the least number of individuals within its neighborhood), and then with probability $\phi$ rewires to a random neighbor of its neighbors who holds the same opinion, or otherwise to a random one selected from the whole population except its nearest neighbors (with probability $1-\phi$).

MP represents the influence of individual acquaintances and also the tendency to adopting the majority opinion among the node's acquaintances. Such preference reflects one's reluctance to be in a minority among friends as well as one's willingness to be in agreement with most friends. MA characterizes the situation in which individuals avoid becoming a minority among their friends and try to form new acquaintanceships with the neighbors of neighbors such that they would have a great chance to become a majority in the MP updating, if presently in the minority. These two rules are in line with the conformism found in human society. In what follows, we will investigate the competing roles of MP and MA in the coevolution of opinions and networks, and concentrate on how the model parameters (i.e., $p$ and $\phi$) affect the evolution.

## III. SIMULATION RESULTS AND DISCUSSIONS

In our simulations, we keep the population size $N$ and the total number of edges $M$ constant during the evolution. Thus the average connectivity $\langle k\rangle = 2M/N$ is fixed. In addition, isolated vertices and duplicate links are not allowed. Initially, individuals are located on a random regular graph (the who-meets-whom relationship is random and homogeneous) and opinions are uniformly assigned to vertices at random. According to our specified update rule, in finite time steps, the system will evidently approach a "consensus state" in which each individual's opinion agrees with the majority of its neighbors. We stop the simulations when this consensus is achieved. At this stage we compute the number of survival opinions and the consensus time as a function of the parameters $p$ and $\phi$, respectively.

In order to gain intuition into the results of our model, we plot the emergent networks in the final stage corresponding to two different $p$ values (see Fig. 1). Clearly, the $p$ value governs the two competing processes in the coevolution. With small $p$ values, the total number of different opinions remains unchanged and individuals with the same opinion are grouped in the same communities, where they have dense connections to others with the same opinion, but rare edges linked to other communities with different opinions [see Fig. 1(a)]. In contrast, for larger $p$ values, all individuals share the same unique opinion and are clustered as a tight group [see Fig. 1(b)]. Based on these results, one can find the expected qualitative behavior of our model: the MP update procedure increases the number of nearest-neighbor vertex pairs with the same opinions, while the MA update consolidates the chance that, without changing opinions, individuals come into agreement with the majority of their neighbors. In particular, when the rewiring process occurs in a triadic manner (i.e., friends of friends), it leads to an increase of the total number of triangles in the network, i.e., the clustering coefficient. We will show that this tendency of redirection to neighbors of neighbors has a recognizable impact on the evolution of opinion. In combination with these two competing update moves, the system will eventually reach a state dependent on the $p$ value: small $p$ values result in the coexistence of different opinions; large $p$ values lead to a uniform opinion among the population.

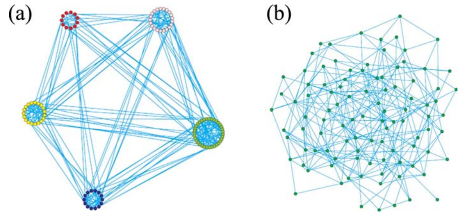

FIG. 1. (Color online) Illustration of the coevolution of opinions and networks. Left panel ($p=0.6, \phi=0.6$): the coexistence of different opinions, in which the individuals are self-organized into communities or groups sharing the same opinion (node color denotes the type of opinion). Right panel ($p=0.8, \phi=0.6$): uniformity of the individual opinions exists, where individuals are clustered in a whole tight group. Parameters: population size $N=100$, average degree $\langle k\rangle=6$, and initial number of opinions $G=5$.

We present the rank-size distribution of opinions in a network in a consensus state in Fig. 2. With small $p$ values, most of the opinions have roughly equal numbers of holders; whereas, with increasing $p$ values, we observe a progressive broadening of the distribution, which clearly presents a heavy tail. As $p$ becomes very large, most opinions go into extinction in the population during the coevolutionary process, but certain opinions dominate until uniformity in opinion is attained. Here, we should point out that, for suitable $p$ values, community structure also emerges along with the co-

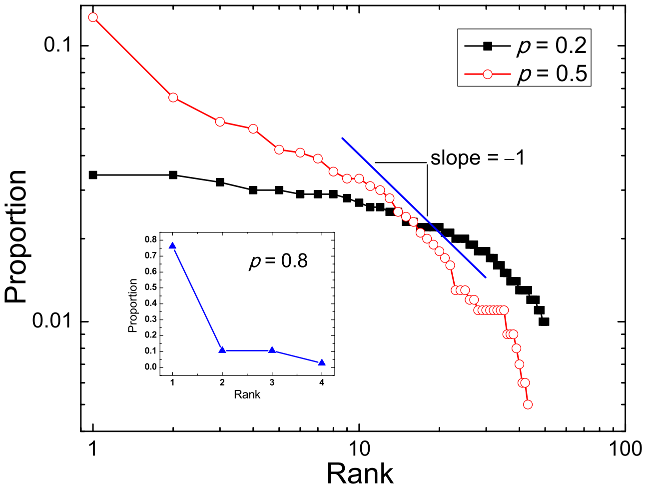

FIG. 2. (Color online) Log-log plot of rank-size distribution, i.e., the rank versus the proportion of individuals holding the same opinion with $p=0.2$ (■) and 0.5 (○). The inset shows the case of large $p=0.8$ (▲). Parameters: $N=1000$, $\langle k\rangle=10$, and $\phi=0.6$.

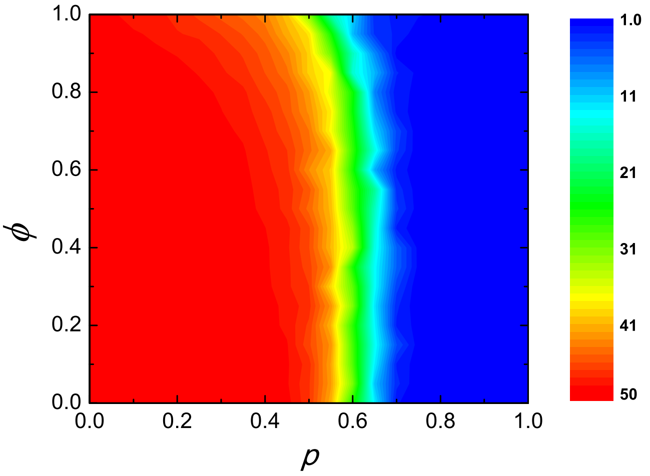

FIG. 3. (Color online) Contour plot of the number of survival opinions as a function of the parameter space $(p,\phi)$. The right color bar indicates the number of opinions. Parameters: $N=1000$, $\langle k\rangle=10$, and $G=50$. Each data point results from an average of 100 independent runs.

evolution. We refer to "community structure" according to the qualitative definition given by Newman—it is groups of network vertices; within these groups there are dense internal links between groups, but between groups there are fewer edges [16]. Interestingly, when $p$ is very small (i.e., $p\to 0$), the community size simply follows the Poisson distribution with mean $N/G$ in the limit of large $N$. As $p$ increases, a few particular opinions successfully become prevalent in the population, indicating that a few giant communities accompanied by some relatively small ones appear. Accordingly, by varying the value of $p$, the system goes from a state in which only small communities exist and almost all opinions exist to one in which there is a giant community of like-minded individuals plus some smaller ones, and most opinions vanish in the population.

We show the final number of opinions as a function of the parameter space $(p,\phi)$ in Fig. 3. For fixed $\phi$, there exists a critical value $p_c(\phi)$ of $p$; below $p_c(\phi)$, a broad variety of opinions are preserved in the population, namely, diversity of individual views is achieved; above $p_c(\phi)$, each individual holds the same opinion, that is, a uniform view is attained in the population. Interestingly, one finds that $p_c(\phi)$ monotonically decreases with increasing $\phi$ value, especially when $\phi$ is large. In addition, for those $p$ values not close to $p_c(\phi)$, the effect of varying $\phi$ values on evolution of opinion vanishes. When $p$ is close to $p_c(\phi)$, a change of $\phi$ plays an important role in the evolution of opinion (see Fig. 3): for fixed $p$, increasing $\phi$ leads to a decrease in the number of surviving opinions. More explicitly, we plot the number of opinions as a function of $\phi$ in Fig. 4. We see that the number of opinions is quickly reduced with $\phi\to 1$. In other words, an increasing tendency of an individual to rewire to neighbors of neighbors quickly reduces the number of survival opinions, and thus is more likely to result in consensus of opinion. Hence, these results demonstrate that the preference toward ordered or random rewiring plays a nontrivial role in evolution of opinion although the parameter $p$ primarily determines the dynamics.

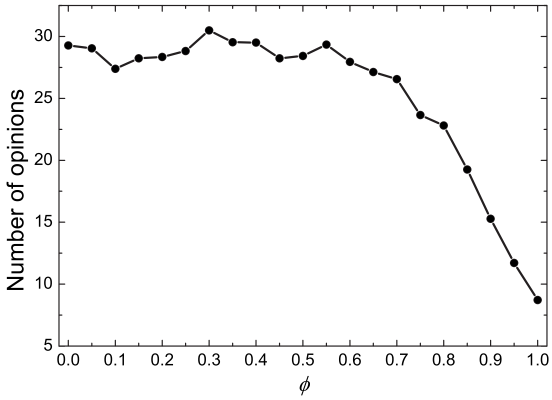

FIG. 4. Final number of opinions versus the parameter $\phi$. Parameters: $N=1000$, $\langle k\rangle=10$, $G=50$, and $p=0.6$. Each data point results from an average of 500 independent runs.

Another quantity of interest is the average consensus time, which is the number of updates needed to reach consensus. We plot the mean consensus time as a function of $p$ with different values of $\phi$ in Fig. 5. Clearly, the consensus time peaks at the critical value of $p$ $(p\approx 0.55)$. For $p<0.55$, the consensus time monotonically increases with increasing $p$ values; while for $p>0.55$, the consensus time drops very quickly as $p$ increases. In accordance with the result shown in Fig. 3, the $p$ value at which the consensus time peaks moves toward the left when $\phi$ is very large (i.e., $\phi\to 1$). This result also indicates that there exists a phase transition around $p_c\approx 0.55$. Furthermore, the system gets absorbed very fast into a uniform state for larger $p$ values. In previous studies of opinion models, finite-size scaling analysis is adopted to give insight into the phase transition in such systems [9]. Moreover, a comprehensive theoretical analysis of these coevolutionary processes was provided recently (see Ref. [15]). We confirm that varying the system size does not qualitatively change the results presented here.

So far, we have presented the main results of our opinion model. Compared with previous works of opinion formation on fixed and evolving networks, our proposed model reflects more realistic scenarios and focuses on a nontrivial network

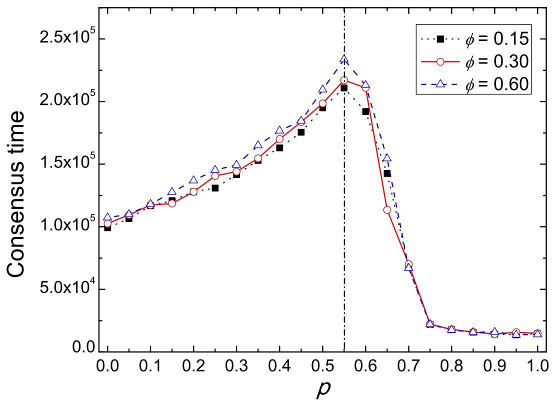

FIG. 5. (Color online) Plot of consensus time as a function of the parameter $p$, with different values of $\phi$=0.15 (■), 0.30 (○), and 0.60 (△). Parameters: $N=1000$, $\langle k\rangle=10$, and $G=50$. Each data point results from an average of 100 independent runs.

rewiring process. Most existing studies allow the network to be disconnected during the coevolutionary process. However, we argue that it is more reasonable to keep the network connected, since this constraint facilitates the investigation of the possible resulting network configurations. Furthermore, we take into account the effect of the tendency to ordered or random rewiring on opinion dynamics. Interestingly, we found that an increasing tendency of an individual to rewire to neighbors of neighbors is more likely to lead to consensus of opinion. This is because, if individuals tend to be grouped with others holding the same opinion, this readily leads to the situation in which some opinions held by the minority are more likely to die out ultimately.

## IV. CONCLUSIONS

We investigated the coevolutionary dynamics of opinions and networks based upon two distinct update moves: MP and MA. These two competing processes capture the characteristics of an individual's opinion change and an individual's neighborhood adjustment, respectively. By varying the model parameter determining the two update moves, the system goes from a diverse world where a broad variety of opinions are present to a uniform one in which everyone shares the same view. Importantly and interestingly, we found that the tendency to rewire to neighbors of neighbors imposes a negative effect on the diversity of opinion. This effect becomes large especially near the critical value of the model parameter.

Recently, many promising research directions are gaining ground, such as the naming game [17], language games [18], and consensus problems [19], which may be viewed as generalized opinion dynamics and formation. The fascinating language dynamics of the struggle between regular and irregular English verbs examined in Ref. [20] could exemplify the significance of this kind of research, both theoretically and empirically. Therefore, it is of much interest and importance to provide a unifying framework to understand these collective dynamics stemming from diverse fields.

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